Writing Mathematics and Assessment in the CDIO project

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> *Assessment is about several things at once. It is not about simple dualities such as grading versus diagnosis. It is about reporting on student's achievements and about teaching them better through expressing to them more clearly the goals of our curricula. It is about measuring students learning and it is about diagnosing specific misunderstandings in order to help students to learn more effectively. It concerns the quality of teachings as well as the quality of learning: it involves us in learning from our student's experiences, and is about changing ourselves as well as our students. It is not only about what students can do; its is also about what it means he or she can do.*
> (Ramsden 1992, p. 182)

Writing mathematics

For many different subjects, in school as well as at universities, the importance of fostering the students to write as part of the learning experience is natural. Subjects like for example English, social science and also science regularly use the writing process as part of their teaching. For the last 10 or 15 years there have been a growing concern about how to create and implement relevant writing assignments also throughout the mathematics curricula. Different reasons such as an increasingly advanced and available technology among learners of mathematics, a growing interest among teachers of mathematics at all levels to learn more about what and how their students learn, and a likewise growing certainty among researchers in mathematics education that assessing knowledge in mathematics is much more than just a written test at the end of a course, have all contributed to a movement about the use of writing assignments in mathematics.

Many authors have discussed the need and importance of students ability to communicate mathematics verbally and literally (NCTM, 2000; Meier & Rishel, 1998; Morgan, 1998; NCTM, 1996, Cowen, 1991; NCTM, 1989). Another continuing ongoing discussion regarding the need for students to take responsibility for their own learning often connect this aim with empowering the students to read, write and discuss mathematics intelligently (Lingefjärd, 2000a; Lingefjärd & Kilpatrick 1998; Cobb, 1986). A third argument is that a realistic view of where students will continue with professional careers after their education is that many of them most likely will be entering into fields where they will be performing scientific, technical, or economical reading and writing, thus they will benefit from writing mathematics in their educational program.

In generally it is not easy to convince teachers of mathematics that they should be part of the writing procedures in the same way as for instance teachers of science, where students

write essays, write problem solving reports, write lab reports, and so forth. Most mathematicians would probably react at first with:

> That is not part of my duty - I'm a mathematician, not a writing instructor!
> For that part, how do you grade a report or an essay?
> Where do I find good writing assignments for my mathematical course?
> (developed from Meier & Rishel, 1998, p. ix.)

Any teacher of mathematics who is reluctant to written assignments should of course try to find out what view she or he has of assessment and students performance. Is assessment just a way of measuring what students know and of expressing what students should learn or can assessment enable students to construct new knowledge from what they know? Should assessment identify those who can or can not perform at a certain occasion or should assessment activities be designed so that all students can begin the activity at some level, even if some students can complete it at a more sophisticated level than others? Is a controlled written paper-and-pencil test the only way to assess students or do you believe in the statement that there are at least three basic sources of assessment information from students: observations, their responses to questions, and examinations of their work.

The communicative aspect of learning mathematics is today so strong and coherent that the Principles and Standards for school mathematics (NCTM 2000) has communication as a strand for all grades. It concludes that:

> Writing is a valuable way of reflecting on and solidifying what one knows, and several kinds of exercises can serve this purpose. For example, teachers can ask students to write down what they have learned about a particular topic or to put together a study guide for a student who was absent and needs to know what is important about the topic. A student who have done a major project or worked on a substantial long-range problem can be asked to compare some of their early work with later work and explain how the later work reflects greater understanding. In these ways, teachers can help students develop skills in mathematical communication that will serve them well both inside and outside the classroom. Using these skills will in turn help students to develop deeper understandings of the mathematical ideas about which they speak, hear, read, and write. (NCTM 2000, p. 352)

Assessing mathematical reports and Taxonomies

A fundamental idea in assessment of students when using written assignments in mathematics, is that assessment and teaching should be integrated. The use of technology as computer programs and graphing calculators naturally affect the evaluation situation and also focus on what we mean by an assessment when technology is involved (Webb 1992). Assessment should naturally use the technology in the same way as it is used in instruction. Beside the possibility to connect teaching and assessment, the technology also offers new possibilities of documentation, visualization, and reporting.

Another, more hidden, objective can very well be to involve the student in the process of evaluation in order to clarify the characteristics of a good mathematical performance. Evaluation may also be seen as a substantial part of the didactical contract that is being established between student and teacher (Brousseau 1997). Through this interplay the students hopefully identify the criteria for a good performance. Further, they can thereby learn what is regarded as an unsatisfactory, good, or very good performance. In many countries, alternative assessment strategies are discussed and tested with the purpose of adopting a more qualitative perspective when assessing students' mathematical performance.

Whenever one wants to examine student's performance in any subject, there is a need for understandable and functional criteria for the evaluation of the achievement. At the first level of examining a written assignment in mathematics, one should ask if:

- The mathematical content – is it correct in terms of notation, figures, diagrams, and conclusions?
- The report – is it written in a language and style that is structured, clear and distinct?
- The problem – is it solved, generalized, explored, and investigated to the limit of all available resources?

A more throughout investigation and grading of a written assignment could be in the terms of the following checklist. It is important to apprehend this checklist as a suggestion of how to appreciate the complexity of a written assignment, not as something that should be followed in every sense when grading.

- Is there a clear introduction, which present the main points of the essay? Is this introduction actually interesting?
- Is there a clear conclusion?
- Is the essay clear organized and easy to follow? Does the student's arguments support the introduction and conclusion?
- Are mathematical items (such as variables) introduced with or without explanations? Do you, as a reader, ever wonder where the mathematical items came from?
- Is there clear indication as to why the student is performing her or his calculations?
- Are the mathematical arguments clear and well stated? Is sufficient detail shown? Are the arguments correct?
- Is the paragraph structure supportive of the student's general discussions?
- How is the spelling, grammar, and punctuation?
  (developed from Meier & Rishel, 1998, p. 34)

Nevertheless if we are using a checklist or not, we are obliged to inform our students what criteria we are using when we are grading their written assignments. This ought to be followed by clear and distinct comments on the papers.

If we look at the mathematical examples in the paper called Assessment and Mathematics Examinations in the CDIO project (Lingefjärd 2000b), we find that the more complicated mathematical knowledge we like to examine among our students, the harder it is to get the examination compressed in a short, closed form. In fact, any attempt to search for and identify a deeper understanding of mathematics calls for interviews, report writing, essays, or written assignments.

Constructing Assessment Tasks

When we are designing a mathematical (or any other subject) assignments, we must ask the following kind of questions:

- What mathematical concept or knowledge am I trying to evaluate with this assignment?
- Is the assigned time length for the assignment appropriate?
- Are the students prepared for this assignment?

- What relevant topics do I value, and which of them is relevant here?
- Does my assignment indicate my priorities to the student?
- Is technology an option or a requirement to accomplish the assignment?

Example 1

Consider the following rather traditional question, borrowed from an analysis course:

> Compute the x and y coordinates of all points on $x^2 + xy + y^2 = 3$ where the curve have horizontal tangents.

In terms of Bloom's or the MATH taxonomy (Lingefjärd, 2000b), this question focus somewhere between knowledge, application, and synthesis. It tests the students' knowledge of and ability to perform implicit differentiation. Further, once the students realize that they need to set the derivative equal to zero and then work their way back to find the actual points, they will have to synthesize their analysis techniques with previous algebra techniques from the gymnasium. Even if there are clever short cuts that can be taken by some one who is good in mathematics, this is actually quite a demanding question. But is it challenging? Does it call for creativity? Is it interesting? Does it apply to students' reality in any sense?

Example 2

Modeling Drug Responses

L-dopa is administered to people suffering from Parkinson's disease to relieve symptoms such as extreme tremors and rigidity. Consider the following set of data on the levels of L-dopa in the blood, in nanograms per milliliter, as a function of time, in minutes after the drug is administered.

t	**0**	**20**	**40**	**60**	**80**	**100**	**120**	**140**	**160**	**180**	**200**	**220**	**240**	**300**	**360**
L	0	300	2700	2950	2600	1550	1100	900	725	600	510	440	300	250	225

a) What function would you use to fit to theses data points? Give at least three different suggestions and discuss the validity and reliability of each of the three functions you propose.

b) When is the level of L-dopa in the blood lower than 10 nanograms per milliliter? Relate your answerers to each of the three models in a).

c) How do you interpret the area under the curves of your function? Calculate the area in all three cases and give a good explanation of how you interpret the meaning of measuring the area and how to understand the result.

How do you value the difference between the task in Example 1 and Example 2? What different mathematical knowledge do they test? How do they differ in relevance for a student? For a creative student in the assessed group, is there a difference in depth for exploration between the examples? Is there a difference between the tasks in terms of the mathematical knowledge one could learn from the problems? Could you think of a way to improve each of the two? How do the problem in Example 2 relate to Bloom's or the MATH taxonomy?

References

Bloom, B.S. (Ed.) (1956). *Taxonomy of educational objectives: The classification of educational goals. Handbook 1: Cognitive domain*. New York: McKay.

Brousseau, G. (1997). *Theory of didactical situations in mathematics: Didactique des mathématiques, 1970-1990* (N. Balacheff, M. Cooper, R. Sutherland, & V. Warfield, Eds. & Trans.). Dordrecht, The Netherlands: Kluwer.

Cobb, P. (1986). Contexts, goals, beliefs, and learning mathematics. *For the Learning of Mathematics,* 6(2), 2-9.

Cowen, C. (1991). Teaching and testing mathematics reading. *American Mathematical Monthly*, 98(1), 50-53.

Lingefjärd, T., & Kilpatrick, J. (1998). Authority and responsibility when learning mathematics in a technology-enhanced environment. In D. Johnson & D. Tinsley (Eds.), *Secondary school mathematics in the world of communication technologies: Learning, teaching and the curriculum* (pp. 233-236). London:

Lingefjärd, T. (2000a). *Mathematical modeling by prospective teachers*. Electronically published doctoral dissertation, University of Georgia. *Can be downloaded from* http://ma-serv.did.gu.se/matematik/thomas.htm

Lingefjärd, T. (2000b). Assessment and Mathematics Examinations in the CDIO project Paper published within the Conception-Design-Implementation-Operation project at Chalmers Technical University.

Meier, J., & Rishel, T. (1998). *Writing in the teaching and learning of mathematics.* Washington DC: Mathematical Association of America.

Morgan, C. (1998). Writing mathematically. The discourse of investigation. London: Falmer press

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (1996). *Communication in mathematics: K - 12 and beyond: 1996 yearboo*k. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Ramsden, P. (1992). *Learning to teach in higher education.* London: Routledge.

Smith, G. H., Wood, L. N., Coupland, M., Stephenson, B., Crawford, K., & Ball, G. (1996). Constructing mathematical examinations to assess a range of knowledge and skills. *International Journal for Mathematical Education in Science and Technology*, *27*(1), 65-77.

Webb, N. L. (1992). Assessment of students' knowledge of mathematics: Steps toward a theory. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 661-683). New York: Macmillan.