

CAN PEER INSTRUCTION IN CALCULUS IMPROVE STUDENT LEARNING?

Mikael Cronhjort

Lars Filipsson

KTH Royal Institute of Technology, Stockholm, Sweden

Maria Weurlander

Karolinska Institutet, Stockholm, Sweden

ABSTRACT

We report on an experiment in which we used Peer Instruction instead of traditional lectures in a Calculus course for beginning engineering students at KTH Royal Institute of Technology. In order to enable evaluation in a controlled experiment setting, we kept the rest of the course – text book, tutorials and examination – unchanged. The student's pre-knowledge was measured by a diagnostic test, and their post-knowledge was measured by the written exam of the course. Our data indicate that the Peer Instruction group learned more than the control group, who had traditional lectures. In questionnaires at the beginning of the course and at the end, we asked for the students' perceptions of Peer Instruction as teaching method and if they had found it useful as a tool for learning calculus. The answers show that the students appreciated being more active and motivated with Peer Instruction, but also that they found the method challenging and somewhat frustrating. A major problem was that the textbook was difficult to read in advance.

KEYWORDS

Peer Instruction, calculus, student learning, students' perceptions

INTRODUCTION

Lectures still dominate much of university mathematics and science teaching, in spite of their many drawbacks. Lectures tend to make the students passive recipients of information and less engaged in building their own knowledge. The use of lectures as the main teaching and learning activity of a course seems to build on the idea that the knowledge of the lecturer can be transmitted somehow to the students, an idea that has been called into question in the last decades [1,2]. Furthermore, students differ in their approaches to learning and sometimes students tend to rely on rote learning where they memorize facts and procedures and subsequently do not reach an understanding of the underlying principles. To help the students reach the proper learning outcomes, it is desirable to use teaching and learning activities that force students to actively engage with the material and spend time practicing to do what they are expected to be able to do after having taken the course. A course designed according to the principles of constructive alignment is more likely to facilitate student success. [1]

There is a well-documented leap between high school and university mathematics [3]. This leap seems to consist of both a gap of content and a clash of cultures. The perception of the subject of mathematics, as expressed in e.g. teaching practice, assessment and national course tests, differ between high school and university. This makes it even more crucial that the course design of the basic university courses help students become active practitioners of mathematics. The well-known problem of students relying too heavily on rote learning and therefore having difficulties reaching a deeper understanding of the subject has inspired teachers to develop course designs where students actively engage in learning. Many efforts have been made to make students more active, e.g. to bring them from imitative to creative reasoning [4], or from procedural to conceptual understanding [5].

In physics, the Force Concept Inventory, FCI, using a conceptual pretest/posttest, showed that the level of conceptual learning is low in many university physics classes. In fact, a large proportion of the students in a traditionally taught class do not gain any deeper conceptual understanding of force and related concepts, even though they pass the course [6].

Peer Instruction, PI, was developed by Eric Mazur in the early 1990's in order to promote conceptual learning and to enhance student activity [7]. Since PI was introduced, the method has proven to be an effective instrument for improving student learning in physics and related subjects [8-11]. With PI, students perform better at procedural as well as conceptual tests [7]. Recent results show that research based instruction may more than double students' learning [8].

The situation in university mathematics today is in many ways similar to the situation in pre-FCI physics. The passing rate in basic calculus and linear algebra courses is in general low. Many teachers feel that the students taking these courses fail to grasp the fundamentals, and that students have difficulties explaining even the simplest examples from first principles. They learn how to plug numbers into ready-made formulas, but conceptual understanding is often lacking, a phenomenon also seen in other subjects [1,2].

Some efforts have been made to teach calculus with PI [10-12]. Even though preliminary data is promising, we have not been able to find any comprehensive studies on this matter.

So far there is no equivalent to the FCI in mathematics – i.e. a universally accepted way to measure the conceptual learning that has taken place during a course – even though attempts have been made. A Calculus Concept Inventory has been proposed [13], but has not yet gained recognition by a greater community of mathematicians and mathematics teachers. Therefore, in our study, we decided to use the course exam to measure the learning in our course. To measure pre-knowledge we used a diagnostic test.

The aim with our study was to investigate, in a controlled experiment setting, if the use of peer instruction instead of traditional lectures would improve student learning in a standard Calculus course, and also investigate student perceptions of peer instruction as a teaching and learning activity.

METHOD

KTH Royal Institute of Technology is a technical university in Stockholm, Sweden, admitting approximately 2 000 beginning engineering students annually. Most of them take the course SF1625, Calculus in one variable, during the first semester. This is a very important course in their education – many later courses build on it, and the concepts introduced are ubiquitous in

engineering applications. Yet quite a large proportion of the students fail the course. In the fall of 2012, the passing rate was only 56%.

Course design

The design of the course involves 21 lectures (90 minutes) and 13 tutorials or problem solving sessions (90 minutes). The assessment of the students is based mainly on a written exam at the end of the course (9 problems, 5 hours), but two quizzes and 4 seminars also contribute toward the grade. Although the students are divided into several lecture groups, the assessment is identical for all students taking the course at a particular time.

In this setting, we performed the following experiment. In one of the lecture groups, we used PI instead of traditional lectures, keeping the rest of the course unchanged, meaning that the students in our group studied the same content, using the same textbook and having the same examination as the students in the other lecture groups. That way we could compare the effect of the different teaching methods.

Before the course began, the students were given a diagnostic test to measure their pre-knowledge [14]. After the course we compared the results of the written exam.

Teaching and learning activities

The PI sessions all followed the same pattern:

- 1) Preparatory reading, i.e. a specific reading assignment for each session.
- 2) A reading report, where the students were asked to describe the most important definitions and theorems of the reading assignment in their own words, was handed in at the beginning of each session. The reading report also had a section where the students were asked to write down what they still had not understood.
- 3) In some cases the lecture began with a brief introduction, aiming at clarifying e.g. terminology or similar minor issues. These clarifications were based on what was written in the students' reading reports, on what they had indicated that they did not yet understand. The lecturer tried to encourage students to more serious reading and better reading reports by awarding clearly described problems encountered in the reading with a clarification.
- 4) Then the main teaching and learning activity began. The lecturer presented a multiple choice question, and the students made up their minds individually on which alternative they thought was right, and answered using clickers. If between 30 and 70 % had the right answer, the students were asked to discuss with those sitting nearby. The lecturer circulated in the theatre, listening to the discussions, and where needed helping them progress in a meaningful direction. If many students still did not arrive at the right answer, the students were asked to compare their answers with other groups nearby. After the peer discussion, students answered again individually by clickers. In general more than 70 %, sometimes more than 90 %, now had the right answer.
- 5) When most of the students had found the right answer, the lecturer in general commented on why the chosen alternative was right, and if necessary on why other alternatives were wrong.
- 6) In a 90 minute lecture we normally had time to present between 6 and 10 clicker questions.

As the contents of the course differ to some extent from the American examples we have found, and we used a Swedish (obviously different) text book, we had to design our own questions.

Furthermore, besides using questions focusing on concepts, we also posed questions dealing with calculations or proofs.

One guideline which we used throughout the course was that the lecturer should not provide answers to questions not yet posed by the students. This implied that our presentations or explanations were always in response to what the students had expressed, either in their reading reports or as answers to the clicker questions.

Research design and data collection

We chose to gather both quantitative and qualitative data in order to investigate both students learning and their perceptions of the PI as a teaching and learning activity. This novel approach enabled us to get a nuanced picture of the usefulness of PI in this particular Calculus course, and adds to our understanding of teaching and learning in Calculus.

Quantitative data

For the quantitative analysis we chose to use data publicly available. Only data concerning entire study programmes were used. The results from the examination were compared to three different measures of the students' pre-knowledge.

Qualitative data

During the course, the students answered an open-ended questionnaire at two occasions, the first about a week after the course start and the second about a week before the end. At both occasions, the questions were the same:

- 1) What are your thoughts on peer instruction as a teaching method?
- 2) Has peer instruction helped you to learn mathematics? In what way? If not, why?

The questionnaire answers were collected by Maria, who is working at a different university and not involved in teaching this particular course. In order to enable us to follow if individuals had changed their minds regarding peer instruction during the course, the students wrote their email address on the questionnaire. The students' answers were made anonymous and their identity treated as confidential.

The answers to these questionnaires were used in a qualitative content analysis, with the aim to explore how students learn mathematics and if peer instruction can help them to learn better than ordinary lectures.

RESULTS

Quantitative results

In order to evaluate the experiment, we compared the results of the course examination to the pre-knowledge for the different study programmes. As it is not evident which is the most appropriate measure of the pre-knowledge, we used three different measures: Results on a diagnostic math test, admission points from upper secondary school, and SweSAT.

The measure of the students' pre-knowledge with the strongest connection to calculus was results from a diagnostic test in mathematics, taken by all students at KTH at the start of the first

year [14]. The average scores of six study programmes on the examination versus their scores on the diagnostic test are shown in figure 1. Linear refers to linear regressions based on the results with PI or with ordinary lectures, respectively. There is no particular reason to believe that there is a linear relation between the scores, but it serves as an approximation as we have results from only a little number of study programmes. As seen in figure 1, our data indicate that the PI group (red) learned more than the control group (blue).

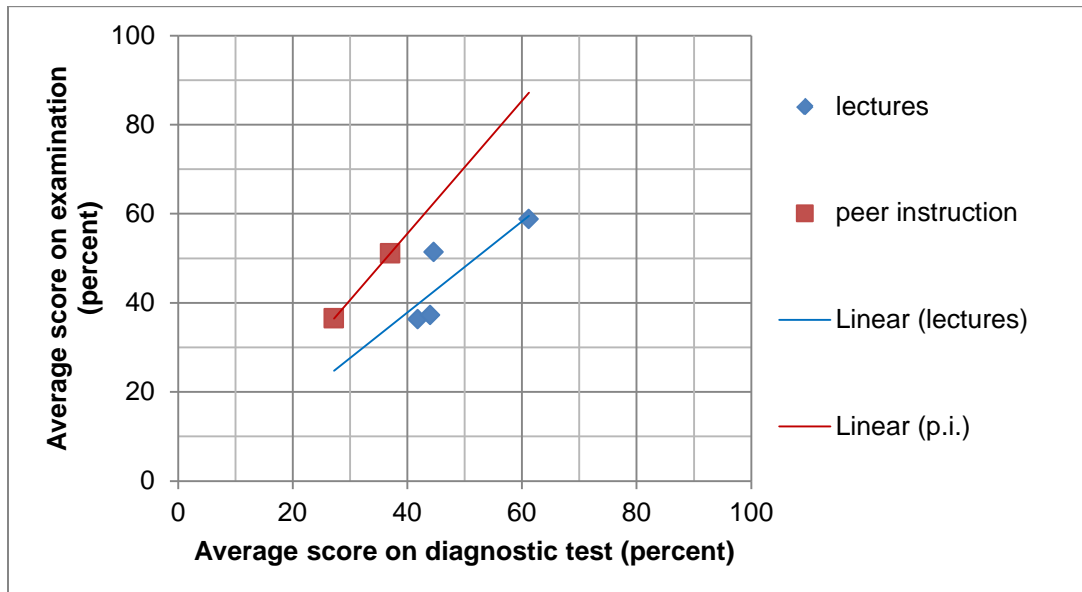


Figure 1. Average scores (%) on the examination in calculus for six study programmes versus the average scores (%) on the initial diagnostic mathematics test. The points correspond to average results for the different study programmes, and the lines represent linear regressions for the study programmes with peer instruction (red) and traditional lectures (blue), respectively. The programmes with peer instruction performed better than expected considering the scores on the diagnostic test.

We also compared the scores on the examination to data from the admission procedure. The majority of students enter study programmes at KTH by merits based on their grades from the upper secondary school. The maximum grade point average is 22.5 and all subjects contribute. The number of points required for admission is determined by the last admitted student. The average scores of the six study programmes on the examination versus the points required for admission are shown in figure 2. The curve of the PI group (red) is above that of the control group (blue), indicating improved learning with PI.

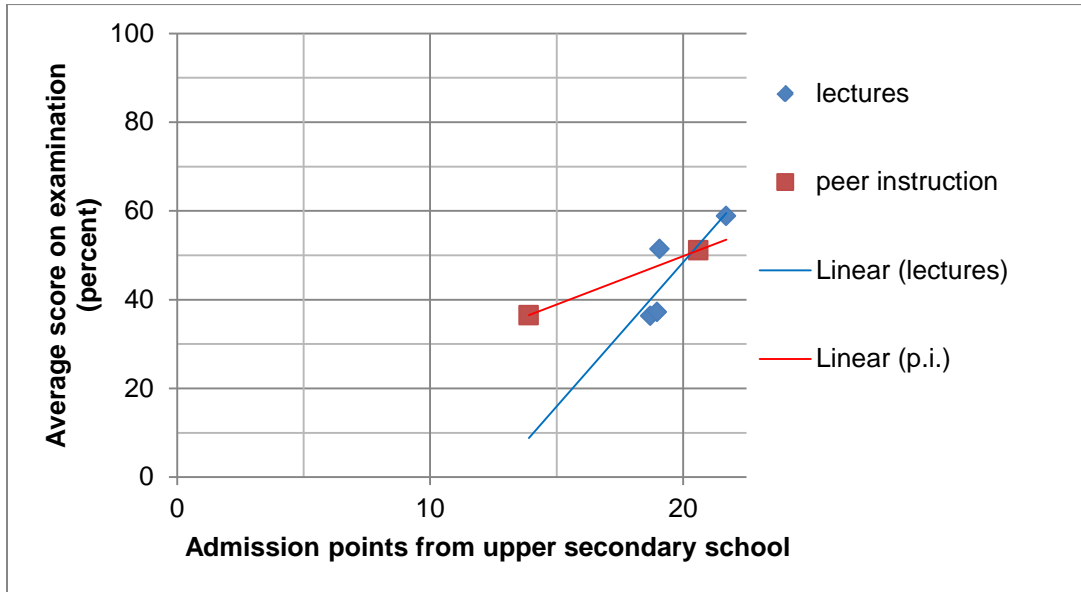


Figure 2. Average scores (%) at examination in calculus versus the admission points required for entering the programme, based on grades from the upper secondary school.

A significant number of students gain admission to KTH through the Swedish Scholastic Assessment Test (SweSAT, Högskoleprovet). The test focuses on mathematics, Swedish and English. For each study programme, there is a requirement limit for SweSAT. The limit is a decimal number between 0 and 2.0, indicating the score of the last admitted student. Figure 3 shows the average scores of the six study programmes on the examination versus the SweSAT requirement limit for admission to the programmes. Also in this case the curve of the PI group is above that of the control group, indicating improved learning.

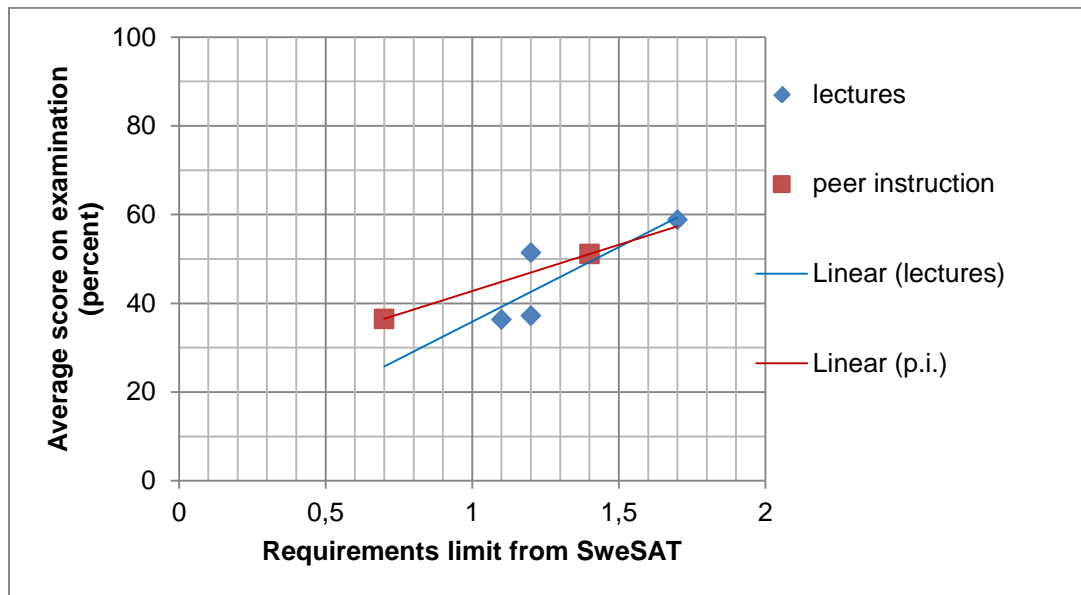


Figure 3. Average scores (%) at examination in calculus versus the SweSAT requirements limit.

Regardless of whether the measure of the pre-knowledge of the study programmes is the results on the diagnostic mathematics test, the grades from the upper secondary school, or SweSAT requirements limits, the students learning calculus by PI demonstrated a better or at least equally good learning. Similar results are obtained if the results on the exam are measured by the percentage students who passed the examination, and if those not completing the exam are included.

Qualitative results

A preliminary analysis of the answers to the open-ended questionnaire show that the students appreciated being more active and motivated with Peer Instruction compared to ordinary lectures. They felt inspired and more engaged during class. The students found the discussions with peers fruitful and many acknowledged the fact that PI required them to read the text book in advance and come prepared to the lectures. But the students also expressed that they found the method challenging and somewhat frustrating. A major problem for the students was that the textbook was difficult to read in advance. Students expressed a concern that they lacked the bigger picture and felt that they only learned the examples that were discussed in class. Furthermore, some students seemed to hold on to a belief that mathematics is best learned when teachers lecture and go through principles and examples that student then review at home.

DISCUSSION AND CONCLUSIONS

This was our first experiment using PI in university mathematics. Our novel approach to combine both quantitative and qualitative data gave a nuanced picture of learning mathematics using PI. We found both strengths and challenges. Our results indicate that the students have learned more, or at least as much as with traditional lectures. In our experiment, the examination was not changed. Except for one theoretical problem, the written exam basically tests the students' problem solving skills and procedural knowledge, and does not at all focus on conceptual understanding. Students can probably be encouraged even more to focus on conceptual understanding with a modified examination. It is still difficult to assess how much. Our findings are in agreement with previous research [9-12], suggesting that PI is an effective teaching and learning method that help students develop a deeper conceptual understanding.

However, we have encountered some difficulties during our experiment with PI. Initially, and to some extent throughout the course, students expressed that learning by this method was somewhat frustrating. Reasons for this might be, that the PI set-up forces the students to engage with the material from day one of the course. They cannot postpone studying. In order to answer the clicker questions they have to reason correctly, using mathematical concepts. Their high school education has not prepared them for this. Also, the PI context makes them realize what level of understanding is required for passing the course. And as the course is difficult, the level required is quite high and they have to struggle hard to reach it.

As lecturers, we also found it frustrating, partially because this teaching method forces the pace to be slowed down as long as the students have not yet understood, and partially because we were made aware of how difficult the students experienced the subject. However, this is also a considerable benefit with PI compared to more traditional teaching methods such as lectures. Knowledge of what students find difficult is useful and important when designing subsequent lectures or courses. A major problem for the students was that the textbook was difficult to read. To overcome this drawback, supplemental course material can be developed to help students understand the text better.

It may also be challenging that teachers and students have new roles in the learning process. Many students expressed an expectation that the teacher was to provide knowledge which the student should acquire. Introducing teaching and learning methods where students are more active might be frustrating to students who are used to more teacher-centered education. It may take a while for students to adapt to a more active student role, and see the teacher more of a facilitator than a lecturer.

REFERENCES

- [1] Biggs J. and Tang C., *Teaching for Quality Learning at University*, 4th edition, Open University Press, Maidenhead, 2011.
- [2] Entwistle N., *Teaching for Understanding at University: Deep Approaches and Distinctive Ways of Thinking*, Palgrave Macmillan, New York, 2009.
- [3] Brandell, G., Hemmi, K. and Thunberg, H., The widening gap – a Swedish perspective, *Mathematics Education Research Journal*, 20:2, 2008, 38-56
- [4] Lithner, J., "A research framework for creative and imitative reasoning," *Educational Studies in Mathematics*, 67, 2008, 255-276
- [5] Engelbrecht, J. Bergsten, C. and Kågesten, O., "Undergraduate students' preference for procedural to conceptual solutions to mathematical problems," *International Journal of Mathematical Education in Science and Technology*, 40:7, 2009, 927-940.
- [6] Halloun, I.A. and Hestenes, D., "The initial knowledge state of collage physics students," *American Journal of Physics*, 53:11, 1985, 1043-48.
- [7] Mazur E., *Peer instruction: a user's manual*, Prentice Hall, Upper Saddle River, New Jersey, 1997.
- [8] Deslauriers L., Schelew E. and Wieman C., "Improved learning in a large-enrollment physics class," *Science* 332, 2011, 862-864.
- [9] Gok T, "The impact of peer instruction on college students' beliefs about physics and conceptual understanding of electricity and magnetism," *International Journal of Science and Mathematics Education*, 10, 2012, 417-436
- [10] Pilzer, S., "Peer instruction in physics and mathematics," *PRIMUS: Problems, Resources and Issues in Mathematics Undergraduate Studies*, 11:2, 2001, 185-192.
- [11] Lucas A., "Using peer instruction and i-clickers to enhance student participation in Calculus," *PRIMUS: Problems, Resources and Issues in Mathematics Undergraduate Studies*, 19:3, 2009, 219-231.
- [12] Miller R.L., Santana-Vega E., and Terrell M.S., "Can good questions and peer discussion improve calculus instruction?," *PRIMUS: Problems, Resources and Issues in Mathematics Undergraduate Studies*, 16:3, 2006, 193-203.
- [13] Epstein J., "Calculus concept inventory," electronic publication in *Proceedings of the National STEM Assessment Conference 2006*, retrieved from [http://www.openwatermedia.com/downloads/STEM\(for-posting\).pdf#page=64](http://www.openwatermedia.com/downloads/STEM(for-posting).pdf#page=64), Jan 25, 2013.

- [14] Brandell L., "Matematikkunskaperna 2012 hos nybörjarna på civilingenjörsprogrammen och andra program vid KTH: Bearbetning av ett förkunskapstest," electronic publication in Swedish on the diagnostic test in mathematics at KTH, retrieved from <http://www.lilahe.com/KTH2012.pdf>, Jan 16, 2013.

BIOGRAPHICAL INFORMATION

Mikael Cronhjort, Ph.D., Educational Developer at KTH Royal Institute of Technology, Stockholm, Sweden. He has a background in theoretical physics and mathematics at KTH. His current field of research is Teaching and Learning in Higher Education with a special focus on mathematics.

Lars Filipsson, Ph.D., associate professor of mathematics at KTH Royal Institute of Technology, Stockholm, Sweden. Director of undergraduate and master's education at KTH School of Science.

Maria Weurlander, Ph.D., Educational Developer and researcher at Karolinska Institutet, Stockholm, Sweden. She has a background in chemistry and biology teaching, Her current research interests is in designing teaching for better learning and how students develop understanding of different subjects in Higher Education.

Corresponding author

Dr. Mikael Cronhjort
KTH Royal Institute of Technology
ECE Lärande, Osquars backe 31
SE-100 44 Stockholm
Sweden
mikaelc@kth.se
Tel: +46-(0)8-7908779



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